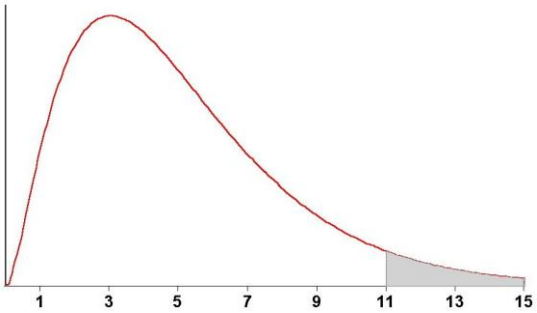
χ^2 (Chi-Square)

- one of the most versatile statistics there is
- can be used in completely different situations than "t" and "z"
- χ^2 is a **skewed** distribution



- Unlike z and t, the tails are not symmetrical.

- X^2 can be used for many different kinds of tests.

We will learn 2 separate kinds of X^2 tests.

Matrix Chi-Square Test
(a.k.a. "Independence" Test)

On a calculator this is just called X^2-Test.

- Compares two qualitative variables, which are usually organized in a **table** (matrix)
- QUESTION: Does the distribution of one variable change from one value to the other variable to another.

EXAMPLES
- Are the colors of M&Ms different in big bags than in small bags?
- In an election, did different ethnic groups vote differently?
- Do different age groups of people access a website in different ways (desktop, laptop, smartphone, etc.)?

The information is generally arranged in a **contingency table** (matrix).
- If you can arrange your data in a table, a matrix chi-square test will probably work.

For example:
Suppose in a TV class there were students at all 5 ILCC centers, in the following distribution:

Center	Male	Female
Algona	5	7
E'burg	3	2
E'ville	4	4
Spenc.	4	7
S.L.	3	3

Does the distribution of men and women vary significantly by center?

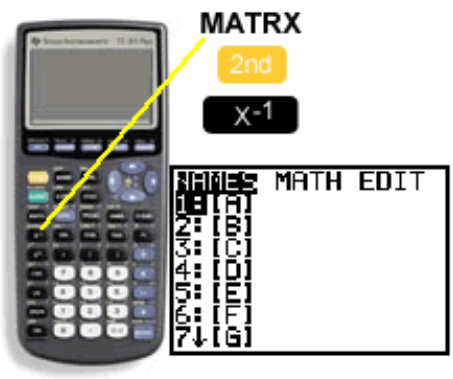
- Our question essentially is—Is the distribution of the columns different from row to row in the table?
- A significant result will mean things **ARE** different from row to row.
- In this case it would mean the male/female distribution varies a lot from center to center.

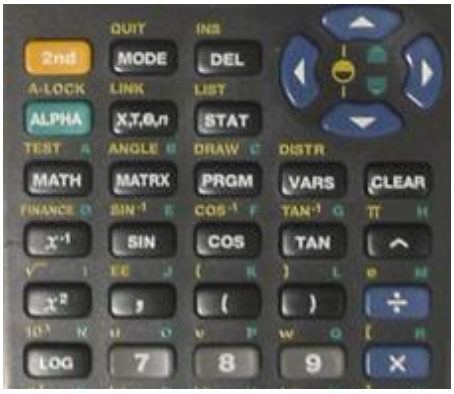
The test process is still the same:
1. Compute a p-value.
2. Compare, and make a decision.

In this problem …

- Since there's no α given in the problem, let's use $\alpha = .05$

1. Enter the observed matrix as [A] in the MATRIX menu.
   - Press MATRX or 2nd and x^{-1}, depending on which TI-83/84 you have.

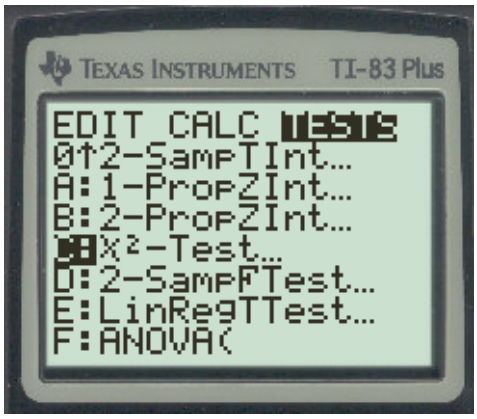




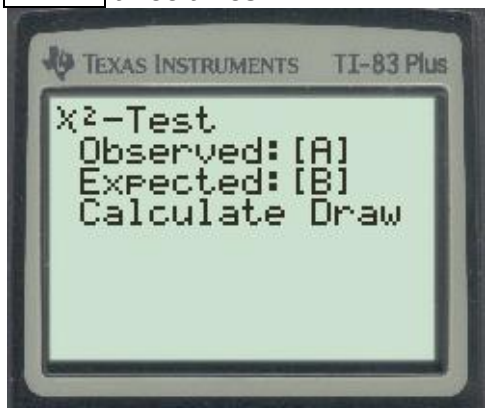
   - Choose "EDIT" (use arrow keys)
   - Choose matrix [A] (just press ENTER)

   - Type the number of rows and columns, pressing ENTER after each.
   - Enter each number, going across each row, and hitting ENTER after each.

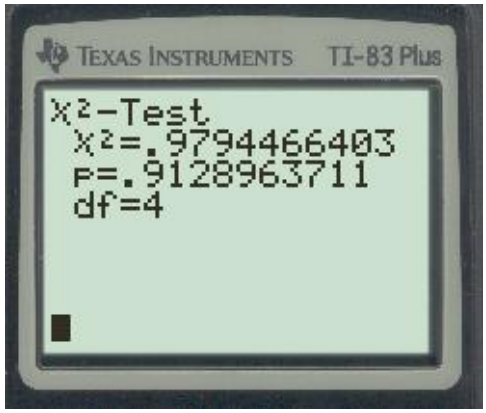
2. Press 2nd and MODE to QUIT back to a blank screen.
3. Go to STAT, then TESTS, and choose X^2-Test (easiest with up arrow)
   (Note on a TI-84 this is "X^2-Test", **not** "X^2-GOF Test")



4. Make sure it says [A] and [B] as the observed and expected matrices. If it does just hit ENTER three times.



5. The read-out will give you χ^2 and the p-value (which is what you care about).



RESULT

- P = .91289
- This is obviously bigger than .05
- NOT significant

Note, it is fairly common to get **very** large p-values with X^2-tests.

Categorical Chi-Square Test
(a.k.a. "Goodness of Fit" Test)

QUESTION:
- Is the distribution of data into various categories different from what is expected?
- Key idea—you have **qualitative** data (characteristics) that can be divided into **more than 2** categories.

EXAMPLES
- ·Are the colors of M&Ms distributed as the company says?
- Is the racial distribution of a community different than it used to be?
- When you roll dice, are the numbers evenly distributed?

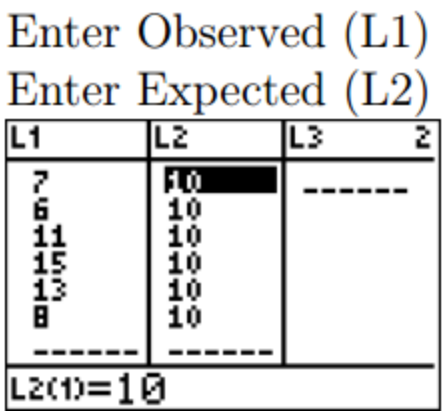
You're comparing what the distribution in different categories **should** be with what it actually is in your sample.

HYPOTHESES:
H_1: The distribution is significantly different from what is expected.
H_0: The distribution is not significantly different from what is expected.

SAMPLE PROBLEM:
You want to know if a die is fair.
You roll it 60 times and get 7 1's, 6 2's, 11 3's, 15 4's, 13 5's, and 8 6's.
At the .10 level of significance can you say the die isn't fair?

This test is not built into the TI-83 (though you can download programs to do it). If you have a TI-84, here's what you do …

Enter the numbers
- Go to STAT → EDIT
- Type the **observed** values in L1.
- Type the **expected** values in L2.



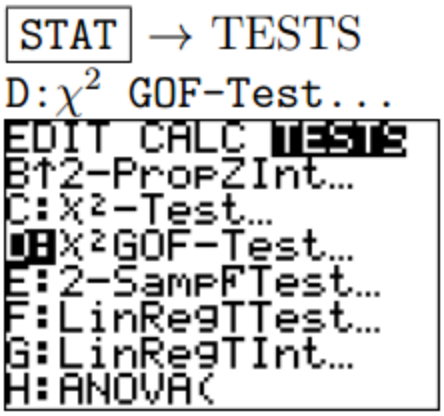
Note that for L2 (expected) you can save time by ...

- If even distribution is expected, take the total divided by the number of categories.
- Otherwise, take each percent times the total.

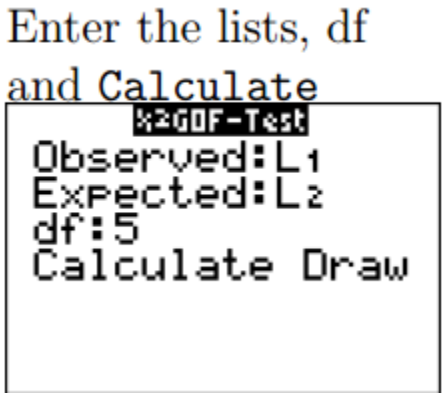
Hit 2nd / MODE to QUIT back to a blank screen.

Do the test

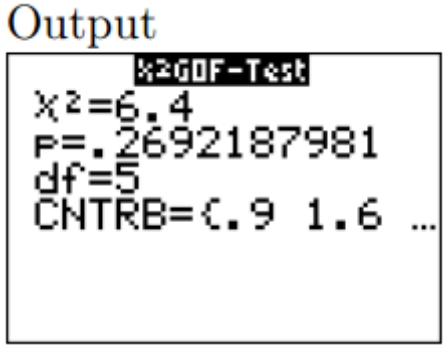
- Go to STAT → TESTS
- Choose choice "D" (you may want to use the up arrow)… X^2GOF-Test



- Make sure Observed says L1 and Expected says L2.
- On the "df" line, enter 1 less than the number of categories.



- As with t and z tests, in the read-out, what you mostly care about is the p-value.



RESULT
.292 > .10 ➔ NOT SIGNIFICANT

EXAMPLE

You think your friend is cheating at cards, so you keep track of which suit all the cards that are played in a hand are. It turns out to be:

- ♦ → 4
- ♥ → 2
- ♣ → 13
- ♠ → 1

You'd normally expect that 25% of all cards would be of each suit. At the .01 level of significance, is this distribution significantly different than should be expected?

Test

STAT → EDIT

L1	L2	L3
4	5	------
2	5	
13	5	
1	5	
------	------	

L2(5) =

2^{nd} → MODE (QUIT)

STAT → TESTS → X^2GOF-Test

```
X²GOF-Test
 Observed:L1
 Expected:L2
 df:3
 Calculate Draw
```

```
X²GOF-Test
 X²=18
 P=4.39849653E-4
 df=3
 CNTRB={.2 1.8 …
```

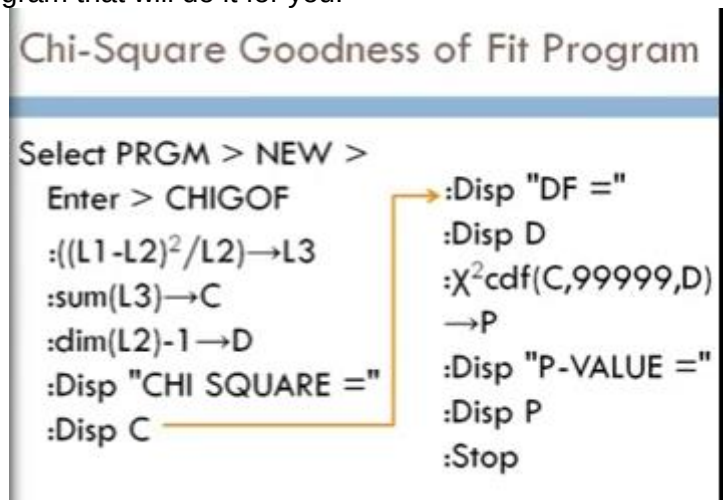
- P =

RESULT:

- .0004398 < .01
- Significant

If you don't have a TI-84 that will do this test …

One option is to enter a program that will do it for you.



These directions are available in the printed notes, and writing this program is explained in detail at this YouTube link … https://www.youtube.com/watch?v=UGEukx2EaEk .

It is also possible to download TI-84 emulators for phones (free) or computers (usually for a fee).

Yet another choice is to go to any of several online X^2 calculators, such as http://vassarstats.net/csfit.html

Cate-gory	Observed Frequency	Expected Frequency	Expected Proportion	Percentage Deviation	Standardized Residuals
A				----	----
B				----	----
C				----	----
D				----	----
E				----	----
F				----	----
G				----	----
H				----	----

Sums:

Observed Frequencies:

Expected Frequencies:

Expected Proportions:

Reset Calculate

[Note that for df=1, the calculated value of chi-square is corrected for continuity.]

[For df=1, this is the uncorrected value of chi-square.]

chi-square =

df =

P =

[P is non-directional]

ONE MORE EXAMPLE:
A teacher wants different types of work to count toward the final grade as follows:

Daily Work	➔	25%
Tests	➔	50%
Project	➔	15%
Class Participation	➔	10%

When points for the term are figured, the actual number of points in each category is:

Daily Work	➔	175
Tests	➔	380
Project	➔	100
Class Participation	➔	75
TOTAL POINTS	**=**	**730**

Was the point distribution significantly different than the teacher said it would be? (Use $\alpha = .05$)

This time it's easiest to take each percent times the total for the expected values.

L1	L2	L3
175	.25*730	------
380	.5*730	
100	.15*730	
75	.1*730	
------	------	

L2(5) =

L1	L2	L3
175	182.5	------
380	365	
100	109.5	
75	73	
------	------	

L2(5) =

Since we have 4 categories, there are 3 degrees of freedom.

X^2GOF-Test
X^2=1.803652968
P=.6141403319
df=3
CNTRB={.308219…

RESULT

.614 > .05, so NOT significant.

The division is roughly the same as what it was supposed to be.