

Example



LOS ANGELES TIMES - February 4, 2005

IQ as a Matter of Life, Death

- Anderson Hawthorne has been convicted of murdering a rival gang member in Los Angeles.
- He has been sentenced to death.
- His lawyer is appealing the sentence, saying that Hawthorne is mentally retarded.
- It is illegal to execute the mentally retarded in the United States.

QUESTION ...

Is he retarded?

- The defendant's IQ has been measured at 71.
- The state used to define 80 or less as "mentally retarded".
- Due to the stigma attached to that label, they recently changed the definition to 70 or lower.
- For IQ, the average is 100, and the standard deviation is 15.

Consider

- What percent of all people have an IQ less than 80?
- ... less than 70?
- ... less than 71?

Problem →

You are a college admissions counselor, considering two students for admission. Jolene got a score of 25 on the ACT, while Roger got a score of 1600 on the new version of the SAT. Who did better on their college placement test?

To answer both of these questions, we will use **standard scores** (or z-scores)

Z-Scores (or standard scores)

- Tell how many standard deviations a score is away from the mean.

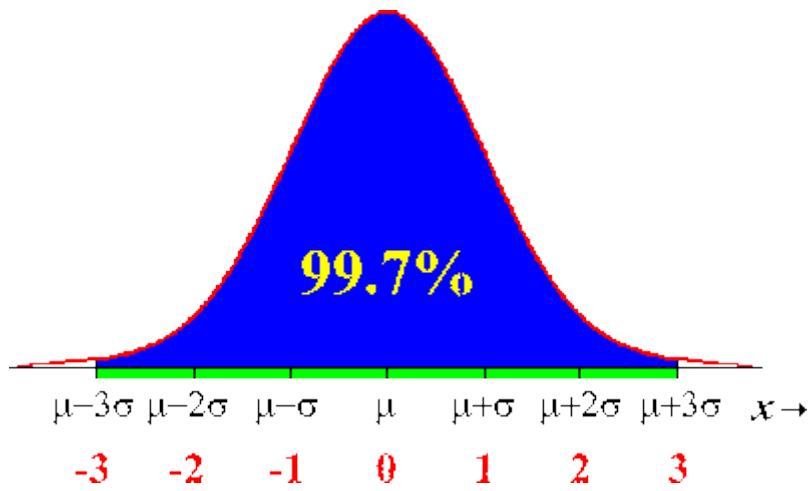
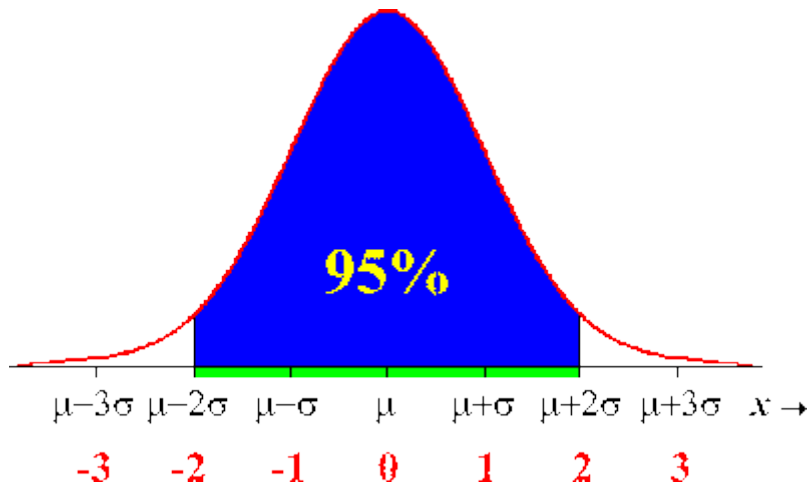
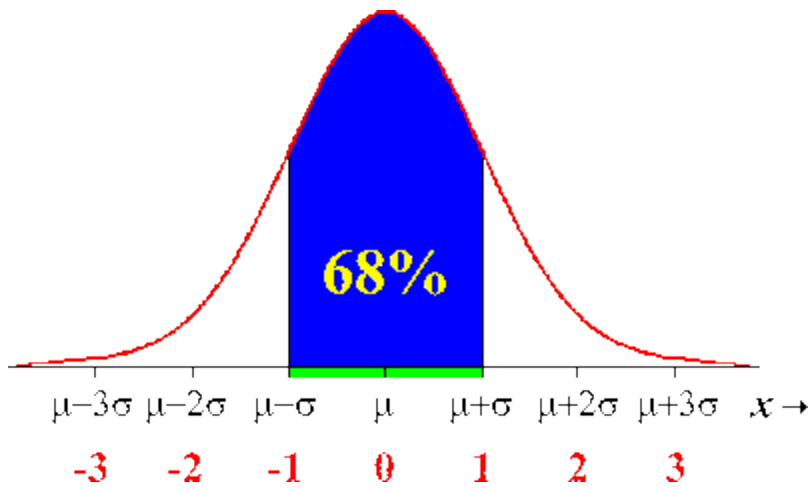
$$Z = \frac{k - \bar{X}}{S} \text{ or } Z = \frac{k - \mu}{\sigma}$$

- A z-score of 0 is average
- **Positive** z-scores are **above** average
- **Negative** z-scores are **below** average

Empirical Rule

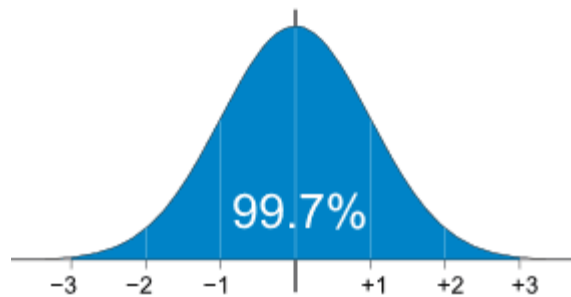
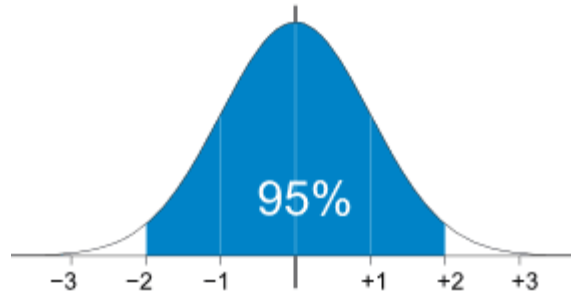
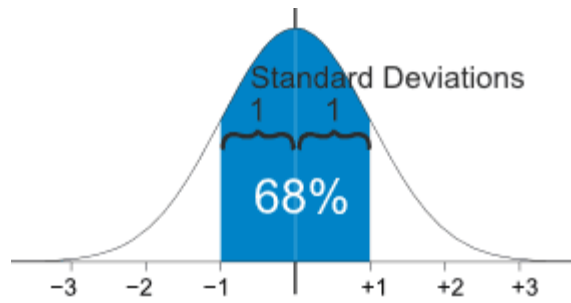
In any distribution that is approximately normal:

- 68% of the data is within 1 S.D. either side of the mean
- 95% of the data is within 2 S.D.s either side of the mean
- 99.7% of the data is within 3 S.D.s either side of the mean



So, ... using the empirical rule:

- ⇒ 68% is between $z = -1$ and $z = 1$
- ⇒ 95% is between $z = -2$ and $z = 2$
- ⇒ 99.7% is between $z = -3$ and $z = 3$



Problem →

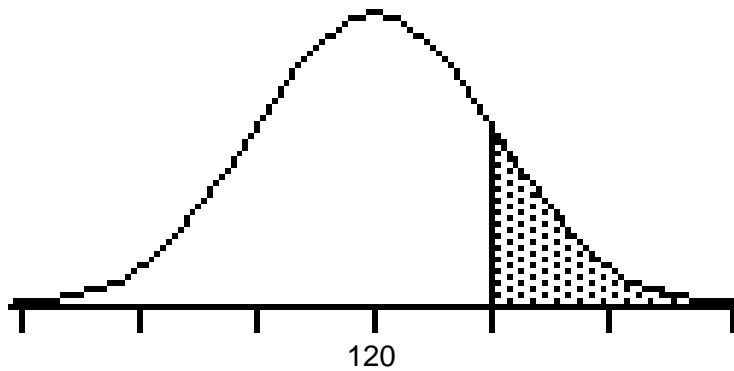
What percent of the population has an IQ above 120?

Area under normal curve

=

Probability of achieving various scores

What we need to do is find the area under the normal curve in the tail beyond an IQ of 120.



Theory: Calculus (antiderivative) gives area under normal curve between two points.


Good news: Somebody's already done it for you.

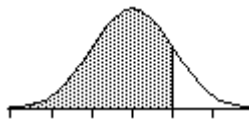
- The results are given as tables in your book.

Useful things to know:

- The **whole** normal curve has an area of **1** (or 100% of the data)
- Each half of the normal curve has an area of **.5** (or 50% of the data)

Your book has two tables –

- the “TAIL” table 

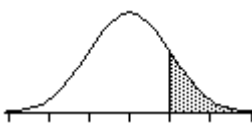
- the “BIG” table 

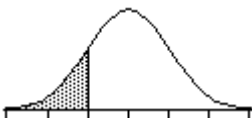
To decide which table to use, it doesn't matter whether z is positive or negative.

What matters is whether you have a big area or a small area.

TYPES OF PROBLEMS

“Tail” Problems

- $z > \text{POSITIVE}$ 

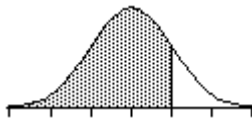
- $z < \text{NEGATIVE}$ 
- Just look up in “tail” table.

- Example: $z > 1.72$

- Example: $z < -2.33$

“Big” (over half) Problems

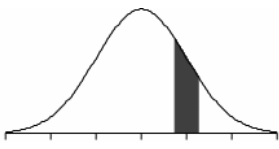
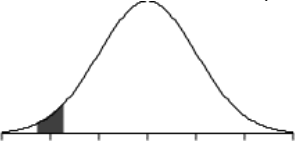
- $z > \text{NEGATIVE}$ 

- $z < \text{POSITIVE}$ 
- Just look up in “big” table.


- Example: $z > -1.23$

- Example: $z < 2.07$

“Same Side” Problems

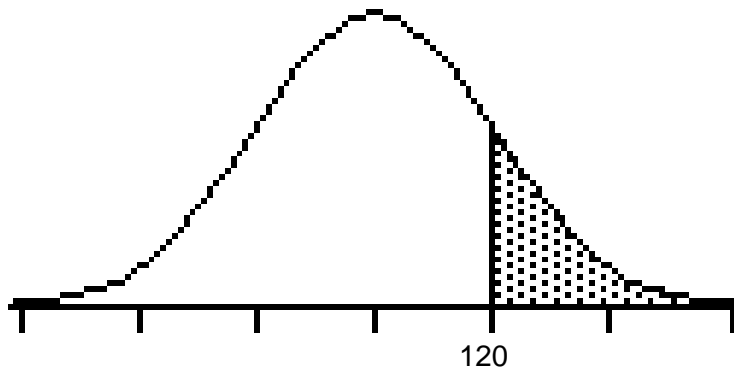
- positive $< z <$ positive  (between two positive numbers)
- negative $< z <$ negative  (between two negative numbers)
- Look up both numbers in the same table.
- Subtract (BIG – SMALL) to get answer.
 - Ex.: $0.45 < z < 1.93$
 - Ex.: $-2.44 < z < -1.60$
 - Ex.: $0 < z < 1.54$

“Both Sides” Problems

- negative $< z <$ positive  (between a negative and a positive)
- Look up both numbers in “tail” table.
- Subtract both tails from 1 ... $1 - \text{FIRST} - \text{SECOND}$
 - Ex.: $-1.28 < z < 0.55$

Back to the original problem...

What we need to do is find the area under the normal curve in the tail beyond an IQ or 120.

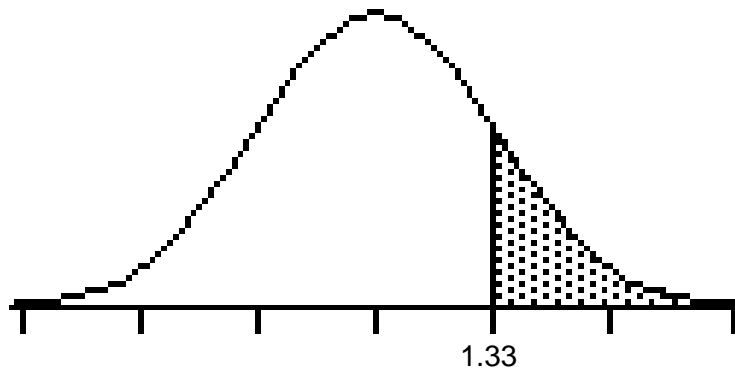


FIRST, find the z-score associated with an IQ of 120. (To do this, you need to know that for IQ $\bar{x} = 120$ and $s = 15$.)

... So
$$z = \frac{20 - 100}{15} = 1.33$$

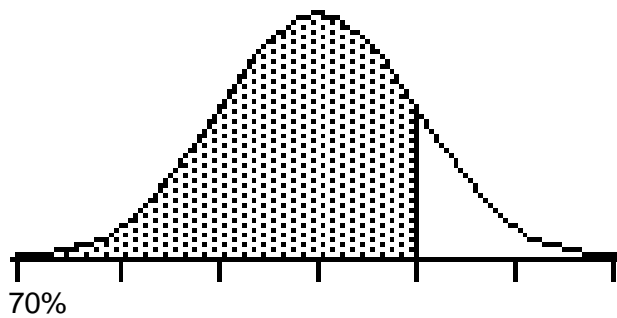
NOW, find the percent of scores that are greater than 1.33 (look up in “tail” table).

- .0918 ... so about 9%.



Sometimes problems are presented backwards.

Find “z” so that 70% of all scores are less than “z”.



Look through the columns in the “big” table for the number closest to .7000 .

- The two closest are .6985 and .7019 .
- .6985 is the closest.
- The associated z-score is **0.52**, which is the answer.

Back to the IQ Example

- For IQ, the average is 100, and the standard deviation is 15.

IQ of 70

$$z = (70-100)/15 = -2.00$$

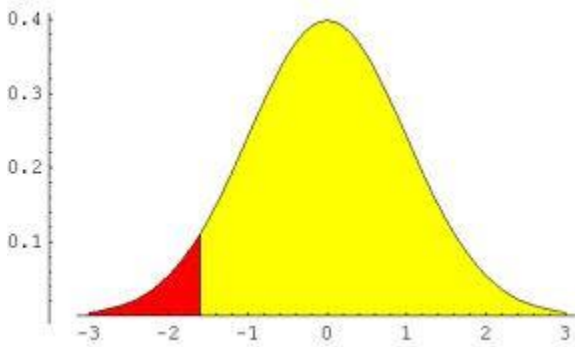
IQ of 80

$$z = (80-100)/15 = -1.33$$

IQ of 71

$$z = (71-100)/15 = -1.93$$

We want the probability z is **LESS** than each of these numbers.



They are all TAIL problems.

IQ of 70

$$P(z < -2.00) = \underline{\mathbf{.0228}}$$

IQ of 80

$$P(z < -1.33) = \underline{\mathbf{.0918}}$$

IQ of 71

$$P(z < -1.93) = \underline{\mathbf{.0268}}$$

Back to the ACT/SAT Example

For ACT ...

$$\text{Mean} = 19.2$$

$$\text{S.D.} = 5.7$$

For new SAT ...

$$\text{Mean} = 1511$$

$$\text{S.D.} = 290$$

JOLENE (25 ACT)

$$z = (25 - 19.2)/5.7 = 1.02$$

ROGER (1600 SAT)

$$z = (1600 - 1511)/290 = 0.31$$

Jolene's z-score is higher, so she did better.