##### Example

newmasthead

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## **IQ as a Matter of Life, Death**

* Anderson Hawthorne has been convicted of murdering a rival gang member in Los Angeles.
* He has been sentenced to death.
* His lawyer is appealing the sentence, saying that Hawthorne is mentally retarded.
* It is illegal to execute the mentally retarded in the United States.

QUESTION …

Is he retarded?

* The defendant’s IQ has been measured at 71.
* The state used to define 80 or less as “mentally retarded”.
* Due to the stigma attached to that label, they recently changed the definition to 70 or lower.
* For IQ, the average is 100, and the standard deviation is 15.

Consider

* What percent of all people have an IQ less than 80?
* … less than 70?
* … less than 71?

**Problem** 🡺

You are a college admissions counselor, considering two students for admission. Jolene got a score of 25 on the ACT, while Roger got a score of 1600 on the new version of the SAT. Who did better on their college placement test?

To answer both of these questions, we will use **standard scores** (or z-scores)

Z-Scores (or standard scores)

* Tell how many standard deviations a score is away from the mean.

 or 

#### A z-score of 0 is average

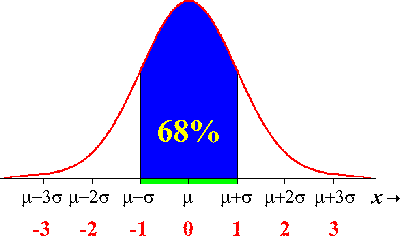
#### **Positive** z-scores are **above** average

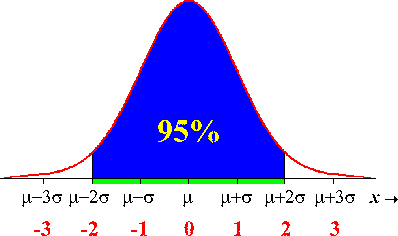
#### **Negative** z-scores are **below** average

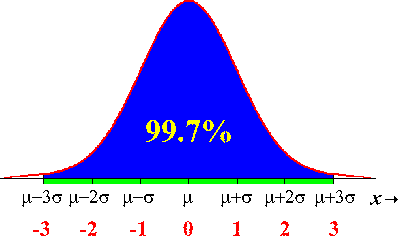
**Empirical Rule**

## In any distribution that is approximately normal:

* 68% of the data is within 1 S.D. either side of the mean
* 95% of the data is within 2 S.D.s either side of the mean
* 99.7% of the data is within 3 S.D.s either side of the mean

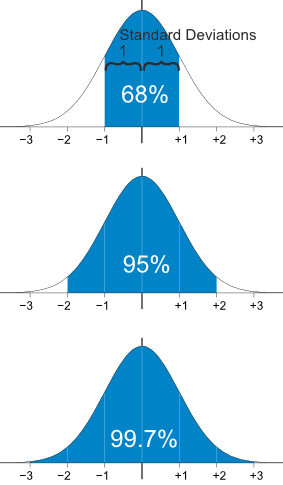






So, … using the empirical rule:

* 68% is between z = -1 and z = 1
* 95% is between z = -2 and z = 2
* 99.7% is between z = -3 and z = 3



**Problem 🡺**

What percent of the population has an IQ above 120?

Area under normal curve

=

Probability of achieving various scores

What we need to do is find the area under the normal curve in the tail beyond an IQ or 120.

big-top-tail

120

**Theory:** Calculus (antiderivative) gives area under normal curve between two points.

**Good news:** Somebody’s already done it for you.

* The results are given as tables in your book.

**Useful things to know:**

* The **whole** normal curve has an area of **1** (or 100% of the data)
* Each half of the normal curve has an area of **.5** (or 50% of the data)

# Your book has two tables –

* the “TAIL” table big-bottom-tail
* the “BIG” table bottom-overhalf

To decide which table to use, it doesn’t matter whether z is positive or negative.  
  
What matters is whether you have a big area or a small area.

TYPES OF PROBLEMS

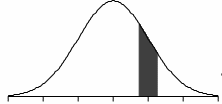
## **“Tail” Problems**

* z > POSITIVE big-top-tail
* z < NEGATIVEbig-bottom-tail
* Just look up in “tail” table.
  + Example: z > 1.72
  + Example: z < -2.33

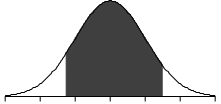
## **“Big” (over half) Problems**

* z > NEGATIVE top-overhalf
* z < POSITIVEbottom-overhalf
* Just look up in “big” table.
  + Example: z > -1.23
  + Example: z < 2.07

## **“Same Side” Problems**

* positive < z < positive (between two positive numbers)
* negative < z < negative (between two negative numbers)
* Look up both numbers in the same table.
* Subtract (BIG – SMALL) to get answer.
  + Ex.: 0.45 < z < 1.93
  + Ex.: -2.44 < z < -1.60
  + Ex.: 0 < z < 1.54

## **“Both Sides” Problems**

* negative < z < positive (between a negative and a positive)
* Look up both numbers in “tail” table.
* Subtract both tails from 1 … 1 – FIRST – SECOND
  + Ex.: -1.28 < z < 0.55

Back to the original problem…

What we need to do is find the area under the normal curve in the tail beyond an IQ or 120.

big-top-tail

120

FIRST, find the z-score associated with a an IQ of 120. (To do this, you need to know that for IQ =120 and s=15.)

… So 

NOW, find the percent of scores that are greater than 1.33 (look up in “tail” table).

* .0918 … so about 9%.

big-top-tail

1.33

Sometimes problems are presented backwards.

Find “z” so that 70% of all scores are less than “z”.

bottom-overhalf

70%

Look through the columns in the “big” table for the number closest to .7000 .

* The two closest are .6985 and .7019 .
* .6985 is the closest.
* The associated z-score is **0.52**, which is the answer.

##### Back to the IQ Example

* For IQ, the average is 100, and the standard deviation is 15.

IQ of 70

z = (70-100)/15 = -2.00

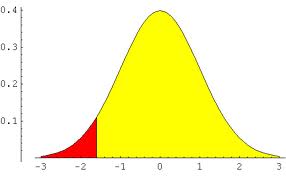
IQ of 80

z = (80-100)/15 = -1.33

IQ of 71

z = (71-100)/15 = -1.93

We want the probability z is **LESS** than each of these numbers.



They are all TAIL problems.

IQ or 70

P(z < -2.00) = **.0228**

IQ of 80

P(z < -1.33) = **.0918**

IQ of 71

P(z < -1.93) = **.0268**

##### Back to the ACT/SAT Example

For ACT …

Mean = 19.2

S.D. = 5.7

For new SAT …

Mean = 1511

S.D. = 290

JOLENE (25 ACT)

z = (25 – 19.2)/5.7 = 1.02

ROGER (1600 SAT)

z = (1600 – 1511)/290 = 0.31

Jolene’s z-score is higher, so she did better.