Non-Parametric Statistics

- Fastest-growing branch of statistics today
- Term comes from the fact that no parameters (no information about the population) is required.

Advantages

- Require very little advance information
- Generally quick & easy to calculate
- Give a quick YES/NO answer to whether a result is significant
- Make no special assumptions (like is the distribution normal)

Disadvantages:

- Not as precise as more traditional statistical tests.
- Does not give a number that can be compared with other results.
- Interpreting results is often different from what you're used to.

There are numerous non-parametric tests, just like there are many traditional tests. We will focus on two of these.

The Sign Test

- Useful for anything that can be easily divided into **two groups**
- Questions a sign test can answer:
  - Is there a significant difference between the **median** of a sample and its expected median?
  - Are more things above a certain number than below it?
  - Did more things go up over time than went down?
  - Are more things in one group than another?

HYPOTHESES:

- H_1: There are significantly more in one group than another.
- H_0: There are roughly the same number in the two groups.

NOTE—There are a lot of ways you can do a sign test. We will do a different method than is explained in the book (which turns it into a z-test).

Critical Value:

- Use the "Sign Test" table (see hand-out)
- Look up "n" (total number in group, excluding any ties) and α.

Test Statistic:

- Count up the number in each group. (Ignore all ties or items equal to the test value.)
- The test statistic is the number in the **SMALLER** group.
- If the same number is in each group, use that number. (Obviously if this is the case it won't be a significant result.)

Interpreting the Results:

- This is **DIFFERENT** from every other test.
- If the number in the smaller group is LESS than the critical value, the result is significant.
- IMPORTANT: You want your number **LESS** than the critical value, not more. The sign test works the opposite of every other test.

EXAMPLE:
Mr. Green thinks it takes less than 10 minutes to get to work. He recorded how long it took him to drive to work each day. The results (in minutes) follow:

8	9.5	7
10.5	6	12
9	9.5	6.5
7	11	11
8	10	7.5
9.5	10	13

Did Mr. Green get to work in under 10 minutes significantly more often than in over 10 minutes? (Use $\alpha = .01$)

Hypotheses:
H_1: < 10 minutes is higher.
H_0: There is not a significant difference between the groups.

Critical Value:
- There are 18 times in the sample, but 2 of them are equal to the test value
- So … use n = 16
- n = 16, $\alpha = .01$
- So critical value = 2

Test Statistic:
- 11 days were < 10 min.
- 5 days were > 10 min.
- 2 days were = 10 min.
- So test statistic = 5

Interpretation:
- For a significant result, we want the test statistic **LESS** than the critical value. Since 5 is **NOT** less than 2, this is **NOT** significant.

EXAMPLE:
Do people's scores go up if they re-take the ACT?
The following table gives people's scores the first and second time they took the ACT.

1st	19	*28*	12	18	21	15	20
2nd	*21*	27	*15*	*20*	21	*17*	*24*
1st	16	*22*	34	29	26	17	13
2nd	*19*	21	*35*	29	*27*	*20*	*14*
1st	20	16	16	*22*	35	27	17
2nd	20	*21*	*18*	21	*36*	*28*	17

- There were 14 times the score went up on the second test.
- There were 3 times the score went down on the second test.
- There were 4 times the score stayed the same.

Critical Value:
- Since the problem doesn't state a level of significance, let's use .05.
- There were 21 in the sample, but 4 were ties, so use n = 17.
- n = 17, $\alpha = .05$ ➔

C.V. is 4.

Test Statistic:
The lower number is 3.

Interpretation:
Since 3 is LESS than 4, this IS a significant result.

Example:
The Department of Chemistry at Bigtime State University is being sued for gender discrimination. The plaintiffs noted that among last year's graduating chemistry majors, there were 50 men and only 34 women. The lawyer for the Bigtime State said there was not a significant difference in the number of male and female chemistry majors. Use a sign test at the .02 level of significance to see who is right.

Critical Value:
- n = 84, α = .02,
- So critical value = 32.

Test Statistic:
- The smaller group has 34 people in it, so 34.

Interpretation:
- Since 34 is NOT less than 32, this is NOT a significant result.
- The differing numbers of male and female majors could be due to chance.

A common question about the sign test is what to do if n > 100.
- One possibility is to convert it into a z-test (which is explained in Chapter 11 of your book)
- An easier alternative is to convert the numbers to percentages (rounded to whole numbers) and compare the percents.

Example:
A software manufacturer conducted a poll to see what type of computer people preferred. They found that 237 people preferred Windows machines, 80 people preferred Macintosh machines, and 74 had no preference. Did significantly more people prefer Windows than Macintosh? (Use α = .10)

237 + 80 = 317 people expressed a preference. (Ignore those who didn't)

Find the percent that preferred each:
- Windows: $\frac{237}{317} = .748$
- Macintosh: $\frac{80}{317} = .252$
- Use 75 and 25.

Critical value
- Use n =100 & α = .01
- C.V. = 36

Test Statistic
- The smaller number is 25

Comparison
- 25 is smaller than 36
- Significant

The Runs Test

QUESTIONS:

- Is the data random?

  OR

- Is there such a pronounced pattern that we should be suspicious of the data?

Hypotheses:

- H_1: The data is NOT random. (There is a problem.)
- H_0: The data is random.

Level of Significance:

For every runs test, we will use $\alpha = .05$, which is the level for the table in the back of your book.

Critical Values:

- Instead of a single value, you will find a **RANGE**.
- To do this:
  1. Divide the data into two groups, based on any obvious characteristic.
  2. Count the number in each group.
  3. Use the larger and smaller numbers to locate the critical range in the hand-out table.
  4. The two numbers are the bottom and top of the critical range.

Test Statistic:

- The test statistic is the number of “runs” in the data.
- A “run” is a group of data that all have the same characteristic.
- Each time you change characteristics, there is another run.

H H H T T T T T H T H
H H T H T H HT T H H
has 11 runs

Interpreting the Results:

- If the number of runs is within the critical range, then the data IS RANDOM (which is technically **not** a significant result).
- The borderline is also considered random.
- If the number of runs is outside the critical range (too many or too few), then the data is NOT RANDOM (which would be a significant result).
- The question will ask if it is random or not.

Example:

(This is simplified from an actual legal case from the 1960s.)

During the Vietnam War, the government held a “draft lottery” to see who would be called for service first. Each possible birth date was assigned a number. The date assigned #1 would be drafted first, the date assigned #2 second, etc. The numbers were assigned by putting balls for each date into a bin and drawing them one at a time from the bin.

A famous lawsuit asserted that the draft numbers were not randomly assigned.
In particular, they asserted that people with birth dates in the last half of the year were more likely to receive low numbers and thus be drafted.

Suppose the first few dates selected were as follows:

Dec. 28	⇒	Nov. 30
Sep. 9	⇒	Jul. 17
Oct. 11	⇒	Nov. 13
Dec. 25	⇒	Jul 4
Aug. 16	⇒	Nov. 28
Sep. 1	⇒	Oct. 22
Dec. 4	⇒	Aug. 7
Oct. 19	⇒	Jul. 30
Jul. 8	⇒	Feb. 11
Jan. 7	⇒	May 7

We have the following data:
- 17 dates July—Dec.
- 3 dates January—June
- 2 runs

Critical Range:
- The larger number is 17
- The smaller number is 3
- So the range is 3 to 8.

Test Statistic:
- The test statistic is the number of runs ➔ 2.

Interpretation of Results:
- Since 2 is NOT in the critical range, the data are NOT random.

(In fact, in the real case, they discovered that they had not properly mixed the bin. Most dates early in the year were toward the bottom of the bin, and most dates toward the end of the year were toward the top.)

Here are some numbers:
(they read across)

4	7	3	5	2
6	9	4	8	6
5	7	3	5	4
8	3	2	1	1

Use the characteristic of “odd” and “even” to decide if the numbers are distributed randomly.

4	<u>7</u>	<u>3</u>	<u>5</u>	***2***
6	<u>9</u>	***4***	***8***	***6***
<u>5</u>	<u>7</u>	<u>3</u>	<u>5</u>	***4***
8	<u>3</u>	***2***	<u>1</u>	<u>1</u>

There are 11 odd numbers and 9 even numbers.

The critical range is 6 to 16.

4	7	3	5	***2***
6	9	***4***	***8***	***6***
5	7	3	5	***4***
8	3	***2***	1	1

There are 10 runs in this data.

Interpretation:

- Since 10 is between 6 and 16, the data is random.